6.2020

Laws of Exponents for Real Numbers

Note

(i) $2^3 \times 2^4$

$= (2\times2\times2)\times(2\times2\times2\times2)$

$= 2^7$

$\therefore a^m \times a^n = a^{m+n}$

(ii) $(2^2)^3$

$= 2^2 \times 2^2 \times 2^2$

$= 2^{2+2+2}$

$= 2^6$

$(a^m)^n = a^{m\times n}$

(iii) $\frac{2^3}{2^2} = \frac{2\times2\times2}{2\times2}$

$= 2$

$= 2^1$

$\therefore \frac{a^m}{a^n} = a^{m-n}$

(iv) $2^2 \times 3^2$

$= 2\times2\times3\times3$

$= 6\times6$

$= 6^2 = (2\times3)^2$

$\therefore a^m \times b^m = (ab)^m$

(v) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

$\therefore \left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4$

(vi) $\left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3$

$= 2^3$

$\therefore \left(\frac{1}{a}\right)^{-n} = \left(\frac{a}{1}\right)^n = a^n$

Note – we use the symbol $\sqrt{x}$, we assume that it is the positive square root of the number x.

we know, $(-2)^2 = (-2) \times (-2) = 4$

$(2)^2 = 2 \times 2 = 4$

So, $\sqrt{4} = 2$, though both '2' and '-2' are square root of 4.

Now,

$\sqrt{a} = (a)^{1/2}$, $\sqrt[3]{a} = (a)^{1/3}$

$\sqrt[4]{a} = (a)^{1/4}$

So, $\boxed{\sqrt[n]{a} = (a)^{1/n}}$

Find

(i) $(5^2)^7 = 5^{2\times 7} = 5^{14}$

(ii) $\dfrac{23^{10}}{23^7} = (23)^{10-7} = (23)^3 = 23^3$

(iii) $4^{\frac{3}{2}} = (4)^{\frac{1}{2}\times 3} = \left(4^{\frac{1}{2}}\right)^3 = (\sqrt{4})^3 = (2)^3 = 8$

OR

$4^{\frac{3}{2}} = (4)^{3\times\frac{1}{2}} = (4^3)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = \sqrt{64} = 8$

(iv) $\sqrt[5]{\sqrt[4]{x^2}} = \left[(x^2)^{\frac{1}{4}}\right]^{\frac{1}{5}} = (x)^{2\times\frac{1}{4}\times\frac{1}{5}}$

$= x^{\frac{2}{4\times 5}}$

$= x^{\frac{1}{10}} = \sqrt[10]{x}$

(v) If $a = 4$, $b = 2$, find $(a^b + b^a)^{-1}$

Ans:- $(a^b + b^a)^{-1}$

$= (4^2 + 2^4)^{-1}$

$= (16 + 16)^{-1}$

$= (32)^{-1}$

$= \left(\frac{32}{1}\right)^{-1}$

$= \left(\frac{1}{32}\right)^{1}$

$= \frac{1}{32}$.

(vi) $\frac{9^{1/7}}{9^{1/6}} = (9)^{\frac{1}{7} - \frac{1}{6}}$

$= (9)^{\frac{6-7}{42}}$

$= (9)^{\frac{-1}{42}}$

$= \left(\frac{9}{1}\right)^{\frac{-1}{42}}$

$= \left(\frac{1}{9}\right)^{\frac{1}{42}}$

$= \frac{(1)^{1/42}}{(9)^{1/42}} = \frac{1}{\sqrt[42]{9}}$.