23.6.2020

Extra Questions on Rationalisation

① If $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$, find (i) 'a' and 'b' (ii) $a^2 + b^2$

A: $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$

$= \frac{(2+\sqrt{3})^2}{(2)^2 - (\sqrt{3})^2}$

$= \frac{(2)^2 + 2(2)(\sqrt{3}) + (\sqrt{3})^2}{4-3}$

$= \frac{4 + 4\sqrt{3} + 3}{1}$

$= \frac{7 + 4\sqrt{3}}{1}$

$= 7 + 4\sqrt{3}$

Now comparing $7 + 4\sqrt{3}$ with $a + b\sqrt{3}$ we get $a = 7$, $b = 4$

(i) The value of $a = 7$ and $b = 4$

(ii) The value of $a^2 + b^2 = (7)^2 + (4)^2$

$= 49 + 16$

$= 65$

② If $x = 5 + 2\sqrt{6}$, find $\sqrt{x} + \frac{1}{\sqrt{x}}$

Soln: we have $x = 5 + 2\sqrt{6}$

Now, $\frac{1}{x} = \frac{1}{(5+2\sqrt{6})} \times \frac{(5-2\sqrt{6})}{(5-2\sqrt{6})}$

$= \frac{(5-2\sqrt{6})}{(5)^2 - (2\sqrt{6})^2}$

$$= \frac{(5-2\sqrt{6})}{(25-24)}$$

$$= \frac{(5-2\sqrt{6})}{1}$$

$$= 5-2\sqrt{6}$$

Rough

$(2\sqrt{6})^2$

$= 2\sqrt{6}\times 2\sqrt{6}$

$= 2\times 2\times \sqrt{6}\times\sqrt{6}$

$= 4\times 6$

$= 24$

$$\therefore \boxed{x = 5+2\sqrt{6}},\ \boxed{\frac{1}{x} = 5-2\sqrt{6}}$$

Now $x+\frac{1}{x} = (5+2\sqrt{6})+(5-2\sqrt{6})$

$$= 5+\cancel{2\sqrt{6}}+5-\cancel{2\sqrt{6}}$$

$$= 10$$

Now, $\boxed{x+\frac{1}{x} = 10}$ — (1)

Rough

$(\sqrt{x})^2 = \sqrt{x}\times\sqrt{x}$

$= x$

$\left(\frac{1}{\sqrt{x}}\right)^2 = \frac{1}{\sqrt{x}}\times\frac{1}{\sqrt{x}}$

$= \frac{1}{x}$

So, we know $(a+b)^2 = a^2+2ab+b^2$

$$\therefore \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2+2(\sqrt{x})\left(\frac{1}{\sqrt{x}}\right)+\left(\frac{1}{\sqrt{x}}\right)^2$$

$$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = x+\frac{2\times\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}+\frac{1}{x}$$

$$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = x+\frac{1}{x}+2$$

$$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = 10+2 \quad \text{(from eq (1))}$$

$$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = 12$$

$$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = (2\sqrt{3})^2$$

2	12
2	6
3	3
	1

$12 = 2^2\times 3$

$\sqrt{12} = 2\sqrt{3}$

$$\Rightarrow \boxed{\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}}$$

$\therefore$ The value of $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$.

6 (3) If $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} = a\sqrt{5} + b\sqrt{2}$ find a & b.

solution:-

$$\frac{2}{\sqrt{5}+\sqrt{3}} = \frac{2}{(\sqrt{5}+\sqrt{3})} \times \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$$

$$= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$= \frac{2(\sqrt{5}-\sqrt{3})}{(5-3)}$$

$$= \frac{\cancel{2}(\sqrt{5}-\sqrt{3})}{\cancel{2}}$$

$$= \sqrt{5} - \sqrt{3}$$

Similarly

$$\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1 \times (\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{(3-2)}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{1}$$

$$= \sqrt{3} - \sqrt{2}$$

Now, the given expression,

$$\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} = (\sqrt{5}-\sqrt{3}) + (\sqrt{3}-\sqrt{2})$$

$$= \sqrt{5} - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} - \sqrt{2}$$

$$= \sqrt{5} - \sqrt{2}$$

$$= \sqrt{5} + (-1)\sqrt{2}$$

~~Now~~

$$= (1)\sqrt{5} + (-1)\sqrt{2}$$

So, $(1)\sqrt{5} + (-1)(\sqrt{2})$ comparing it with $a\sqrt{5} + b\sqrt{2}$ we get

$a = (1)$ & $b = (-1)$