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## Extra Problems Based on Rationalisation

① If $x = \sqrt{5} + 2$, find (i) $\left(x^2 + \frac{1}{x^2}\right)$

(ii) $\left(x - \frac{1}{x}\right)$

(iii) $\left(x - \frac{1}{x}\right)^2$

Solution:-

If $x = \sqrt{5}+2$

$$\therefore \frac{1}{x} = \frac{1}{\sqrt{5}+2}$$

$$= \frac{1}{(\sqrt{5}+2)} \times \frac{(\sqrt{5}-2)}{(\sqrt{5}-2)}$$

$$= \frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{(\sqrt{5}-2)}{(5-4)}$$

$$= \frac{(\sqrt{5}-2)}{1}$$

$$= \sqrt{5}-2$$

$$\therefore x + \frac{1}{x} = (\sqrt{5}+2) + (\sqrt{5}-2)$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{5} + \cancel{2} + \sqrt{5} - \cancel{2}$$

$$\Rightarrow x + \frac{1}{x} = 2\sqrt{5} \longrightarrow ①$$

Doing whole square on Both Sides of eq ①,

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = \left(2\sqrt{5}\right)^2$$

$$\Rightarrow (x)^2 + \left(2 \times x \times \frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = (2\sqrt{5} \times 2\sqrt{5})$$

$\Rightarrow x^2 + \frac{2x}{x} + \frac{1}{x^2} = (4 \times 5)$

~~$\Rightarrow x^2 + 2 + \frac{1}{x^2} = 20$~~

$\Rightarrow x^2 + \frac{1}{x^2} = 20 - 2$

$\Rightarrow \boxed{x^2 + \frac{1}{x^2} = 18}$

$\therefore$ The value of $x^2 + \frac{1}{x^2} = 18$.

(ii) $x - \frac{1}{x} = (\sqrt{5} + 2) - (\sqrt{5} - 2)$

$\Rightarrow \left(x - \frac{1}{x}\right) = \cancel{\sqrt{5}} + 2 - \cancel{\sqrt{5}} + 2$

$\Rightarrow \left(x - \frac{1}{x}\right) = 4$

$\therefore$ The value of $\left(x - \frac{1}{x}\right)$ is 4.

(iii) Value of $\left(x - \frac{1}{x}\right)^2 = (4)^2 = 16$.

② If $a = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$ and $b = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$, find the value of $a^2 + b^2 - 5ab$.

Solution:-

$a = \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})} \times \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} - \sqrt{2})}$

$\Rightarrow a = \frac{(\sqrt{3} - \sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2}$

$\Rightarrow a = \frac{(\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2}{(3 - 2)}$

$\Rightarrow a = \frac{3 - 2\sqrt{6} + 2}{1}$

$\Rightarrow a = \frac{5 - 2\sqrt{6}}{1}$

$\Rightarrow \boxed{a = 5 - 2\sqrt{6}}$ — (i)

Now

$b = \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})}$

$\Rightarrow b = \frac{(\sqrt{3} + \sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2}$

$\Rightarrow b = \frac{(\sqrt{3})^2 + 2(\sqrt{3})(\sqrt{2}) + (\sqrt{2})^2}{(3 - 2)}$

$\Rightarrow b = \frac{3 + 2\sqrt{6} + 2}{1}$

$\Rightarrow b = \frac{5 + 2\sqrt{6}}{1}$

$\Rightarrow \boxed{b = 5 + 2\sqrt{6}}$ — (ii)

Now, $a^2 = (5 - 2\sqrt{6})^2$

$\Rightarrow a^2 = (5)^2 - 2(5)(2\sqrt{6}) + (2\sqrt{6})^2$

$\Rightarrow a^2 = 25 - 20\sqrt{6} + (2\sqrt{6} \times 2\sqrt{6})$

$\Rightarrow a^2 = 25 - 20\sqrt{6} + 24$

$\Rightarrow \boxed{a^2 = 49 - 20\sqrt{6}}$

$b^2 = (5+2\sqrt{6})^2$

$\Rightarrow b^2 = (5)^2 + 2(5)(2\sqrt{6}) + (2\sqrt{6})^2$

$\Rightarrow b^2 = 25 + 20\sqrt{6} + 24$

$\Rightarrow \boxed{b^2 = 49 + 20\sqrt{6}}$

Now,

$5ab = 5(5-2\sqrt{6})(5+2\sqrt{6})$

$\Rightarrow 5ab = 5[(5)^2 - (2\sqrt{6})^2]$

$\Rightarrow 5ab = 5[25 - 24]$

$\Rightarrow 5ab = 5(1)$

$\Rightarrow \boxed{5ab = 5}$

$\therefore$ The value of $a^2 + b^2 - 5ab$

$= (49 - 20\sqrt{6}) + (49 + 20\sqrt{6}) - (5)$

$= 49 - \cancel{20\sqrt{6}} + 49 + \cancel{20\sqrt{6}} - 5$

$= 49 + 49 - 5$

$= 93$

③ Find the value of '$x$', if $2^{9x} \div 2^{2x} = \sqrt[7]{2^{14}}$

Solution,

$2^{9x} \div 2^{2x} = \sqrt[7]{2^{14}}$

$\Rightarrow (2)^{9x-2x} = (2^{14})^{\frac{1}{7}}$

$\Rightarrow (2)^{7x} = (2)^{14\times\frac{1}{7}}$

$\Rightarrow (2)^{7x} = (2)^{\frac{14}{7}}$

$\Rightarrow (2)^{7x} = (2)^2$

$\Rightarrow (2)^{7x} = (2)^{2}$

Since the powers have same bases on both sides, their respective exponents must be equal

$\Rightarrow 7x = 2$

$\Rightarrow x = \frac{2}{7}$

$\therefore$ The value of $x$ is $\frac{2}{7}$.

(4) If $a^2 - b^2 = 1$, find the value of

$\left[\frac{(x)^a}{(x)^b}\right]^{a+b}$

Solution – Given expression is $= \left[\frac{x^a}{x^b}\right]^{a+b}$

$= \left[(x)^{a-b}\right]^{a+b}$

$= [x]^{(a-b)\times(a+b)}$

$= (x)^{a^2-b^2}$

$= (x)^1$ [given that $a^2 - b^2 = 1$]

$= x$