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C.W (Extra Problems)

(1) Prove that,

$$\frac{2^{31}+2^{30}+2^{29}}{2^{32}+2^{31}-2^{30}} = \frac{7}{10}$$

Solution:-

L.H.S

$$= \frac{2^{31}+2^{30}+2^{29}}{2^{32}+2^{31}-2^{30}}$$

$$= \frac{(2^2\cdot 2^{29})+(2^1\cdot 2^{29})+(1\cdot 2^{29})}{(2^2\cdot 2^{30})+(2^1\cdot 2^{30})-(1\cdot 2^{30})}$$

$$= \frac{2^{29}\cdot(2^2+2^1+1)}{2^{30}\cdot(2^2+2^1-1)}$$

$$= \frac{\cancel{2^{29}}\times(4+2+1)}{2^1\times\cancel{2^{29}}\times(4+2-1)}$$

$$= \frac{1\times(7)}{2\times(5)}$$

$$= \frac{7}{10}$$

R.H.S

$$= \frac{7}{10}$$

$\therefore$ L.H.S = R.H.S

It is verified that $\frac{2^{31}+2^{30}+2^{29}}{2^{32}+2^{31}-2^{30}} = \frac{7}{10}$

⑤ Find the value of x if
$a = \frac{2^{x-1}}{2^{x-2}}$, $b = \frac{2^{-x}}{2^{x+1}}$ and $(a-b) = 0$.

Solution:- $a = \frac{2^{x-1}}{2^{x-2}} = (2)^{(x-1)-(x-2)}$

$= (2)^{x-1-x+2}$

$= (2)^{x-x-1+2}$

$= (2)^{0+1}$

$= (2)^{1}$

$= 2$

similarly,
$b = \frac{2^{-x}}{2^{x+1}} = (2)^{(-x)-(x+1)}$

$= (2)^{-x-x-1}$

$= (2)^{-2x-1}$

$= (2)^{-2x-1}$

Now given that
$a - b = 0$

$\Rightarrow (2) - (2)^{-2x-1} = 0$

$\Rightarrow (2) = 0 + (2)^{-2x-1}$

$\Rightarrow (2)^{1} = (2)^{-2x-1}$

Since the powers have same Bases on Both sides, their respective exponents must be equal.

$\Rightarrow 1 = -2x-1$

$\Rightarrow 1+1 = -2x$

$\Rightarrow$

$\Rightarrow 2 = -2x$

$\Rightarrow 2 = 1 - 2x$

$\Rightarrow \dfrac{2}{-2} = x$

$\Rightarrow (-1) = x$

$\therefore$ The value of x is -1.

(3) Simplify the expression:-

$$\frac{2\sqrt{30}}{\sqrt{6}} - \frac{3\sqrt{140}}{\sqrt{28}} + \frac{\sqrt{55}}{\sqrt{99}}$$

Solution-
$$\frac{2\sqrt{30}}{\sqrt{6}} - \frac{3\sqrt{140}}{\sqrt{28}} + \frac{\sqrt{55}}{\sqrt{99}}$$

$$= \frac{2\sqrt{5}\times\cancel{\sqrt{6}}}{\cancel{\sqrt{6}}} - \frac{3\cancel{\sqrt{4}}\times\sqrt{5}\times\cancel{\sqrt{7}}}{\cancel{\sqrt{7}}\times\cancel{\sqrt{4}}} + \frac{\sqrt{5}\times\cancel{\sqrt{11}}}{\sqrt{9}\times\cancel{\sqrt{11}}}$$

$$= 2\sqrt{5} - 3\sqrt{5} + \frac{\sqrt{5}}{\sqrt{9}}$$

$$= -1\sqrt{5} + \frac{\sqrt{5}}{\sqrt{9}}$$

$$= \frac{-1\sqrt{5}}{1} + \frac{\sqrt{5}}{3} \quad [\because \sqrt{9} = 3]$$

$$= \frac{3(-1\sqrt{5}) + 1(\sqrt{5})}{3}$$

$$= \frac{-3\sqrt{5} + \sqrt{5}}{3}$$

$$= \frac{-2\sqrt{5}}{3}$$

Rough

$\sqrt{30} = \sqrt{5\times6}$
$= \sqrt{5}\times\sqrt{6}$

$\sqrt{140} = \sqrt{4\times35}$
$= \sqrt{4\times7\times5}$
$= \sqrt{4}\times\sqrt{7}\times\sqrt{5}$

$\sqrt{55} = \sqrt{5\times11}$
$= \sqrt{5}\times\sqrt{11}$

$\sqrt{99} = \sqrt{9\times11}$
$= \sqrt{9}\times\sqrt{11}$

(4) Simplify :-

(1) $\sqrt[3]{27} - 7\sqrt[3]{216} + 10\sqrt[6]{64} + \sqrt{121}$

Solution

$\sqrt[3]{27} - 7\sqrt[3]{216} + 10\sqrt[6]{64} + \sqrt{121}$

$= (27)^{\frac{1}{3}} - 7(216)^{\frac{1}{3}} + 10(64)^{\frac{1}{6}} + (121)^{\frac{1}{2}}$

$= (3^3)^{\frac{1}{3}} - 7(6^3)^{\frac{1}{3}} + 10(2^6)^{\frac{1}{6}} + (11^2)^{\frac{1}{2}}$

$= (3)^{3\times\frac{1}{3}} - 7(6)^{3\times\frac{1}{3}} + 10(2)^{6\times\frac{1}{6}} + (11)^{2\times\frac{1}{2}}$

$= 3^1 - 7(6)^1 + 10(2)^1 + (11)^1$

$= 3 - 42 + 20 + 11$

$= -39 + 31$

$= -8$

(5) Simplify $\sqrt{a^3 b^4} \cdot \sqrt[3]{a^4 b^3}$

Solution $\sqrt{a^3 b^4} \cdot \sqrt[3]{a^4 b^3}$

$= (a^3 b^4)^{\frac{1}{2}} \cdot (a^4 b^3)^{\frac{1}{3}}$

$= \left[(a^3)^{\frac{1}{2}} \cdot (b^4)^{\frac{1}{2}}\right] \cdot \left[(a^4)^{\frac{1}{3}} (b^3)^{\frac{1}{3}}\right]$

$= a^{\frac{3}{2}} \cdot b^{\frac{4}{2}} \cdot a^{\frac{4}{3}} \, b^{\frac{3}{3}}$

$= a^{\frac{3}{2}} \times b^2 \times a^{\frac{4}{3}} \times b^1$

$= a^{\frac{3}{2}+\frac{4}{3}} \times b^{2+1}$

$= a^{\frac{9+8}{6}} \times b^3$

$= a^{\frac{17}{6}} \times b^3$

$\therefore$ The answer is $a^{\frac{17}{6}} \times b^3$